STATIONARY WAVES
(Waves and Vibration)

e-content for B.Sc Physics (Honours)
B.Sc Part-I
Paper-I

Dr. Ayan Mukherjee,
Assistant Professor,
Department of Physics,
Ram Ratan Singh College, Mokama.
Patliputra University, Patna

STATIONARY WAVES

1. A sonometer wire of length 0.5 m is stretched by a weight of 5 kg. The fundamental frequency of vibration is 100 Hz. Determine the linear density of material of wire.

Given :

$l = 0.5$ m

$T = Mg = 5 \times 9.8$ N

$n = 100$ Hz

To Find :

$m = ?$

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore \quad 100 = \frac{1}{2 \times 0.5}\sqrt{\frac{5 \times 9.8}{m}} = \sqrt{\frac{49}{m}}$$

Squaring both sides, we get

$$(100)^2 = \frac{49}{m}$$

$$\therefore \quad m = \frac{49}{(100)^2} = 0.0049 \text{ kg/m}$$

$$\therefore \quad m = 0.0049 \text{ kg/m}$$

2. A sonometer wire 100 cm long produces a resonance with a tuning fork. When its length is decreased by 10 cm, 8 beats per second are heard. Find the frequency of tuning fork.

Given :

$l_1 = 100$ cm

$l_2 = 100 - 10 = 90$ cm

$N = n_1 \sim n_2 = 8 s^{-1}$

To Find :

$n_1 = ?$

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Since, $n_1 \propto \frac{1}{l_1}$ and $n_2 \propto \frac{1}{l_2}$

$$\therefore \quad \frac{n_1}{n_2} = \frac{l_2}{l_1}$$

$$\therefore \quad n_1 l_1 = n_2 l_2$$

$$\therefore \quad 100\, n_1 = 90\, n_2$$

$$\therefore \quad n_2 = \frac{100}{90} n_1$$

$$= \frac{10}{9} n_1$$

$\because \quad l_1 > l_2$ then $n_2 > n_1$

$$\therefore \quad n_2 - n_1 = 8$$

$$\therefore \quad \frac{10}{9} n_1 - n_1 = 8$$

$$n_1\left(\frac{10}{9} - 1\right) = 8$$

$$\therefore \quad n_1 = 9 \times 8 = 72 \text{ Hz}$$

$$\therefore \quad n_1 = n = 72 \text{ Hz}$$

3. Two wires of the same material and having the same radius have their fundamental frequencies in the ratio 1 : 2, and tension in the ratio 1 : 8 compare ratio of their lengths.

Given :

$$\frac{n_1}{n_2} = \frac{1}{2}$$

$$\frac{T_1}{T_2} = \frac{1}{8}$$

To Find :

$$\frac{l_1}{l_2} = ?$$

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore \quad n_1 = \frac{1}{2l_1}\sqrt{\frac{T_1}{m}}$$

$$\therefore \quad l_1 = \frac{1}{2n_1}\sqrt{\frac{T_1}{m}} \quad \text{... (i)}$$

For second case,

$$n_2 = \frac{1}{2l_2}\sqrt{\frac{T_2}{m}}$$

$$\therefore \quad l_2 = \frac{1}{2n_2}\sqrt{\frac{T_2}{m}} \quad \text{... (ii)}$$

[m is constant because both wires are made of same material]

Divide (i) by (ii)

$$\frac{l_1}{l_2} = \frac{2n_2}{2n_1}\sqrt{\frac{T_1}{m}\times\frac{m}{T_2}}$$

$$= \frac{n_2}{n_1}\sqrt{\frac{T_1}{T_2}}$$

$$= \left(\frac{2}{1}\right)\sqrt{\frac{1}{8}}$$

$$\therefore \quad \frac{l_1}{l_2} = 0.707 : 1$$

4. A wire, 1 m long and weighing 2 g, will be in resonance with a frequency of 300 Hz. Find tension on stretching the wire.

Given :

$l = 1$ m

$M = 2$ gm $= 2 \times 10^{-3}$ kg

$n = 300$ Hz

To Find :

$T = ?$

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Solution :

$$\frac{M}{l} = \frac{2\times10^{-3}}{1}$$

$$= 2 \times 10^{-3} \text{ kg/m}$$

$$\text{Now, } n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore \quad 300 = \frac{1}{2\times l}\sqrt{\frac{T}{2\times10^{-3}}}$$

$$600 = \sqrt{\frac{T}{2\times10^{-3}}}$$

$$\therefore \quad (600)^2 = \frac{T}{2\times10^{-3}}$$

$$\therefore \quad T = 36 \times 10^4 \times 2 \times 10^{-3}$$

$$\therefore \quad T = 720 \text{ N}$$

5. A stretched sonometer wire is in unison with a tuning fork, when the length is increased by 4 %, the number of beats heard per second is 6. find the frequency of the fork.

Given :

$l_2 = 1.04\, l_1$

$\therefore \quad \frac{l_2}{l_1} = 1.04,$

$N = 6 s^{-1}$

To Find :

$n_1 = ?$

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Since $l_2 > l_1$

then $n_1 > n_2$

$\therefore\ n_1 - n_2 = 6$ Hz

$\therefore\ n_2 = n_1 - 6$

For two wires of same material

$n_1 l_1 = n_2 l_2$

But , $n_2 = n_1 - 6$

and $l_2 = 1.04\, l_1$

$\therefore\ n_1 l_1 = (n_1 - 6)(1.04\, l_1)$

$\therefore\ n_1 = (n_1 - 6)(1.04)$

$\therefore\ 1.04\, n_1 - n_1 = 6.24$

$\therefore\ 0.04\, n_1 = 6.24$

$$\therefore\ n_1 = \frac{6.24}{0.04}$$

$\therefore\ n_1 = 156$ Hz

6. **The speed of a transverse wave along a uniform metal wire, when it is under a tension of 1000 g wt. is 68 m/s. If the density of metal is 7900 kg/m³. Find the area of cross section of the wire.**

Given :

$T = 1000$ g wt.

$= 1000 \times 10^{-3}$ kg wt.

$= 1 \times 9.8$ N $= 9.8$ N

$v = 68$ m/s

$\rho = 7900$ kg/m³

To Find :

$A = ?$

Formula :

$$v = \sqrt{\frac{T}{m}}$$

Solution :

Since mass of the wire,

$M = V\rho = Al\rho$

Also,

$$m = \frac{M}{l} = \frac{Al\rho}{l}$$

$\therefore\ m = A\rho$

$$\text{Now, } v = \sqrt{\frac{T}{m}}$$

$$\therefore\ v = \sqrt{\frac{T}{A\rho}}$$

$$\therefore\ v^2 = \frac{T}{A\rho}$$

$$\therefore\ A = \frac{T}{v^2\rho}$$

$$A = \frac{9.8}{(68)^2 \times 7900}$$

$\therefore\ A = 2.683 \times 10^{-7}$ m²

7. **A transverse wave is produced on a streched string 0.7 m long and fixed at its ends. Find speed of transverse wave, when it vibrates, emitting the second overtone of frequency 300 Hz.**

Given :

$l = 0.7$ m

$n = 300$ Hz

To Find :

$v = ?$

Formula :

$v = n\lambda$

Solution :

$v = n\lambda$

In 2nd overtone, 3 loops are formed

$$\therefore\ l = \frac{3}{2}\lambda$$

$$\therefore\ \lambda = \frac{2l}{3}$$

$$\text{Now, } v = n\lambda$$

$$v = n\left(\frac{2l}{3}\right)$$

$$v = 300\left(\frac{2}{3}\times 0.7\right)$$

$$\therefore \quad v = 140 \text{ m/s}$$

8. A uniform wire under tension, is fixed at its ends. If the ratio of tension in the wire to the square of its length is 360 dyne/cm² and fundamental frequency of vibration of wire is 300 Hz. Find its linear density.

Given :

$$\frac{T}{l^2} = 360 \text{ dyne/cm}^2$$

$$n = 300 \text{ Hz}$$

To Find :

$$m = ?$$

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$n^2 = \frac{1}{4l^2}\cdot\frac{T}{m}$$

$$\therefore \quad m = \frac{1}{4n^2}\cdot\left(\frac{T}{l^2}\right)$$

$$m = \frac{1}{4\times(300)^2}\times\frac{360}{1}$$

$$m = \frac{90}{90000}$$

$$m = 10^{-3}$$

$$\therefore \quad m = 10^{-3} \text{ g/cm}$$

$$m = 10^{-4} \text{ kg/m}$$

9. A wire is in unison with a fork of frequency 250 Hz, when streched by a weight hanging vertically. On immersing the weight in water, the wire produces ten beats per second with the same fork. Calculate density of material of weight,

Given :

When wire is stretched by a weight hanging vertically, n_1 = 250 Hz,

Frequency of wire when the weight is immersed in water producing 10 beats per second = n_2

$$\therefore \quad n_2 = n_1 - 10 = 250 - 10 = 240 \text{ Hz}$$

$$\rho_w = 1 \text{ g/cc}$$

To Find :

$$\rho = ?$$

Formula :

$$\frac{n_1}{n_2} = \sqrt{\frac{\rho}{\rho - 1}}$$

Solution :

$$\frac{n_1}{n_2} = \sqrt{\frac{\rho}{\rho - 1}}$$

$$\frac{250}{240} = \sqrt{\frac{\rho}{\rho - 1}}$$

$$\therefore \quad \frac{25}{24} = \sqrt{\frac{\rho}{\rho - 1}}$$

Squaring both sides,

$$\frac{625}{576} = \frac{\rho}{\rho - 1}$$

$$\therefore \quad 625(\rho - 1) = 576\rho$$

$$\therefore \quad 625\rho - 576\rho = 625$$

$$\therefore \quad 49\rho = 625$$

$$\therefore \quad \rho = \frac{625}{49}$$

$$\therefore \quad \rho = 12.76 \text{ g/cm}^3$$

10. Two simple harmonic progressive waves are represented by

$$y_1 = 2 \sin 2\pi\left(100t - \frac{x}{60}\right) \text{cm and}$$

$$y_2 = 2 \sin 2\pi\left(100t + \frac{x}{60}\right) \text{cm.}$$

The waves combine to from a stationary wave.

Find :

i) amplitude at antinode

ii) distance between adjacent node and antinode

iii) loop length

iv) wave velocity

Given :

$$y_1 = 2 \sin 2\pi \left(100t - \frac{x}{60}\right) \text{cm}$$

$$y_2 = 2 \sin 2\pi \left(100t + \frac{x}{60}\right) \text{cm}$$

To Find :

i) $R = ?$

ii) $\frac{\lambda}{4} = ?$

iii) $l = ?$

iv) $v = ?$

Formula :

$$y = R \sin 2\pi nt$$

where, $R = 2A \cos\left(\frac{2\pi x}{\lambda}\right)$

Solution :

i) Resultantant equation of wave is given by

$$y = y_1 + y_2$$

$$= 2 \sin 2\pi \left(100t - \frac{x}{60}\right) + 2 \sin 2\pi \left(100t + \frac{x}{60}\right)$$

$$\therefore \quad y = 2 \times 2 \sin 2\pi\ (100\ t) \cos 2\pi \left(\frac{x}{60}\right)$$

$$\therefore \quad y = 4 \cos\left(\frac{2\pi x}{60}\right) \sin 2\pi\ (100\ t) \quad \text{...(i)}$$

Comparing above equation with,

$y = R \sin (2\pi t)$

We get,

$$4 \cos\left(\frac{2\pi x}{60}\right) = R$$

But, $R = 2\pi \cos\left(\frac{2\pi R}{\lambda}\right)$

$\therefore \quad \lambda = 60$ cm

Amplitude at antinode is maximum value of R.

i.e, R is maximum when $\cos\left(\frac{2\pi x}{\lambda}\right) = 1$

$\therefore \quad R = 4 \times 1 = 4$ cm

ii) $\lambda = 60$ cm

$\therefore \quad \frac{\lambda}{4} = \frac{60}{4} = 15$ cm

$\therefore$ Distance between successive node and antinode = 15 cm

iii) l = length of loop

$$= \frac{\lambda}{2} = \frac{60}{2}$$

$\therefore \quad l = 30$ cm

iv) v = wave velocity $= n\lambda$

$\therefore \quad v = n\lambda$

From equation (i), we get,

$n = 100$ Hz

$\lambda = 60$ cm

$\therefore \quad v = n\lambda$

$v = 100 \times 60$

$v = 6000$ cm/s

$\therefore \quad v = 60$ m/s

11. The equation of a standing wave is given by y = 0.02 cos (πx) sin (100 π t) m. Find the amplitude of either wave interfering, wavelength, time period, frequency and wave velocity of interfering waves.

Given :

$y = 0.02 \cos(\pi x) \sin(100 \pi t)$

To Find :

$A = ?$

$\lambda = ?$

$T = ?$

$n = ?$

$v = ?$

Formula :

$y = R \sin 2\pi nt$

where,

$R = 2A \cos\frac{2\pi x}{\lambda}$

Calculation :

$y = 0.02 \cos(\pi x) \sin(100 \pi t)$

$\therefore \quad y = 0.02 \cos\left(\frac{2\pi x}{2}\right) \sin[2\pi (50)t]$

Comparing the given equation with,

$y = R \sin 2\pi nt$

where,

$R = 2A \cos\frac{2\pi x}{\lambda}$

We get,

A = amplitudeof interfering waves

= 0.01 m

λ = Wavelength of interfering waves

= 2m

n = Frequency of interfering waves

= 50 Hz

T = Time period interfering waves

$= \frac{1}{n} = \frac{1}{50} = 0.025$

v = Velocity of interfering waves

$= n\lambda = 50 \times 2$

$\therefore \quad v = 100 \text{ m/s}$

12. In Melde's experiment, find weight added in the pan when number of loops on the string changes from 4 to 2. If initial tension on the string is 1960 dyne and mass of the pan in one gram.

Given :

$p_1 = 4$

$p_2 = 2$

$M_0 = 1 \text{ g}$

$T_1 = (M_0 + M_1)g$

$= 1960 \text{ dyne}$

$T_2 = (M_0 + M_2)g$

To Find :

$M_2 = ?$

Formula :

$T_1 p_1^2 = T_2 p_2^2$

Solution :

$T_1 p_1^2 = T_2 p_2^2$

$T_2 = \frac{T_1 p_1^2}{p_2^2}$

$= \frac{(M_0 + M_1)g(4)^2}{(2)^2}$

$\therefore \quad T_2 = \frac{16(M_0 + M_1)g}{4}$

$= \frac{1960 \times 16}{4}$

$T_2 = 4 \times 1960$

But $(M_0 + M_2)g = T_2$

$\therefore \quad (M_0 + M_2)g = 4 \times 1960$

$(1 + M_2)\, 980 = 4 \times 1960$

$\therefore \quad (M_2 + 1) = 4 \times 2$

$\therefore \quad M_2 + 1 = 8$

$\therefore \quad M_2 = 8 - 1$

$\therefore \quad M_2 = 7 \text{ g wt}$

13. In Melde's experiemnt, fork was arranged in parallel position and 6 loops were formed along a length of 7.2 m when stretched by a weight of 10 g. If mass of the string is 14.4×10^{-2} g, find the freqeuncy of tuning fork.

Given :

p = 6

L = 7.2 m

M = 10 g

= 10×10^{-3}

= 10^{-2} kg

M′ = 14.4×10^{-2} g

= 14.4×10^{-5} kg

To Find :

N = ?

Formula :

$$n = \frac{P}{2L}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{P}{2L}\sqrt{\frac{T}{m}}$$

For parallel position, frequency of fork is given by

N = 2n

$$\therefore \quad N = \frac{2.P}{2L}\sqrt{\frac{T}{m}}$$

$$N = \frac{P}{L}\sqrt{\frac{T}{m}} \qquad \text{...(i)}$$

$$\text{Now, } m = \frac{M'}{l}$$

$$m = \frac{14.4\times10^{-5}}{7.2}$$

$\therefore$ m = 2×10^{-5} kg/m

Substituting the values in (i), we get,

$$N = \frac{6}{7.2}\sqrt{\frac{10^{-2}\times9.8}{2\times10^{-5}}}$$

$$N = \frac{6}{7.2}\sqrt{49\times10^{2}}$$

$$N = \frac{6\times7\times10}{7.2} = \frac{70}{1.2}$$

$\therefore$ N = 58.33 Hz

14. Find the frequency of fifth overtone of an air column vibrating in a pipe closed at one end, length of pipe is 42.10 cm and speed of sound in air at room temperature is 350 m/s.
[Inner diameter of pipe is 3.5 cm]

Given :

l = 42.10cm

= 42.10×10^{-2} m

= 0.4210 m

v = 350 m/s

d = 3.5 cm = 3.5×10^{-2} m

= 0.035 m

To Find :

Frequecy of fifth overtone n_5 = ?

Formula :

Fundamental frequency in air column closed at one end is given by

$$n = \frac{V}{4L}$$

Solution :

L = l + 0.3 d

= 0.4210 + 0.3 × 0.035

= 0.42 10 + 0.0105

$\therefore$ L = 0.4315 m

$$\text{Now, } n = \frac{V}{4L}$$

$$n = \frac{350}{4(0.4315)}$$

$$\therefore \quad n = \frac{350}{4\times0.4315}$$

n = 202.78 Hz

P^{th} overtone is given by $n_p = (2p + 1)n$

$\therefore$ Frequencey of fifth overtone is,

$\therefore$ n_5 = (2 × 5 + 1)n

$\therefore$ n_5 = 11 n

= 11 × 202.78

= 2230.59 Hz

$\therefore$ n_5 = 2230.59 Hz

15. Two organ pipes, open at both ends, are sounded together and 5 beats are heard per second. The length of shorter pipe is 0.25 m. find the length of the otherpipe. (Given : velocity of sound in air = 350 m/s, end orrection at one end = 0.015 m same for both pipes)

Given :

$l_1 = 0.25$ m

$v = 350$ m/s

$n_1 - n_2 = 5$

$e = 0.015$ m same for both pipes.

To Find :

$l_2 = ?$

Formula :

$$n = \frac{V}{2L}$$

Solution :

Fundamental frequency of organ pipe open at both ends

$$n = \frac{v}{2L}$$

where

$L = l + e$

since $L_1 < L_2$ then $n_1 > n_2$

$n_1 - n_2 = 5$ ... (i)

Since pipe is open at both ends hence end correction is (2e)

$\therefore$ correct length of 1st pipe

$$L_1 = l_1 + 2e$$

$$= 0.25 + 2 \times 0.015$$

$$L_1 = 0.25 + 0.03 = 0.28 \text{ m}$$

$$\therefore \quad n_1 = \frac{v}{2L_1}$$

$$= \frac{350}{2 \times 0.28}$$

$$= \frac{350}{0.56} = 625 \text{ Hz}$$

$$\therefore \quad n_1 = 625 \text{ Hz}$$

Using (i), we get,

$$625 - n_2 = 5$$

$$\therefore \quad n_2 = 620 \text{ Hz}$$

$$\text{Also, } n_2 = \frac{v}{2L_2}$$

$$\therefore \quad 620 = \frac{350}{2L_2}$$

$$\therefore \quad 2L_2 = \frac{350}{620}$$

$$\therefore \quad L_2 = \frac{350}{2 \times 620}$$

$$L_2 = \frac{35}{124} = 0.2823 \text{ m}$$

$$\because \quad L_2 = l_2 + 2e$$

$$\therefore \quad l_2 = L_2 - 2e$$

$$l_2 = 0.2823 - 0.03$$

$$\therefore \quad l_2 = 0.2523 \text{ m}$$

16. The fundamental frequency of a pipe closed at one end is unison with the third overtone of an open pipe. Calculate the ratio of their lengths of air column.

Given :

$n_o = n_c$

where

n_o = frequency of third overtone of open pipe

n_c = fundametal freqeuncy of closed pipe

To Find :

$$\frac{L_c}{L_o} = ?$$

Formula :

$$n = \frac{V}{2L}$$

Solution :

3rd overtone of open pipe is given by

$$n_o = 4\left(\frac{v}{2L_o}\right)$$

Fundamental frequency of closed pipe at one end is given by,

$$n_c = \frac{v}{4L_c}$$

$$\because \quad n_c = n_o$$

$$\therefore \quad \frac{v}{4L_c} = 4\left(\frac{v}{2L_o}\right)$$

$$\therefore \quad \frac{L_o}{L_c} = 8$$

$$\therefore \quad \frac{L_c}{L_o} = \frac{1}{8}$$

$$\therefore \quad \frac{L_c}{L_o} = 1 : 8$$

17. Show that for a pipe open at both ends the end correction is

$$\mathbf{e = \frac{n_2 l_2 - n_1 l_1}{2(n_1 - n_2)}}.$$

Solution :

Let,

l_1 and l_2 = Vibrating lengths of pipe

n_1 and n_2 = Resonating freqeuncy

v = Velocity of sound in air

e = End correction

i) For the first resonance

$$n_1 = \frac{v}{2(l_1 + 2e)}$$

$$\therefore \quad v = 2n_1 (l_1 + 2e) \quad ...(i)$$

ii) For the second resonance

$$n_2 = \frac{v}{2(l_2 + 2e)}$$

$$\therefore \quad v = 2n_2 (l_2 + 2e) \quad ...(ii)$$

iii) From (i) and (ii) we get,

$$\therefore \quad n_1(l_1 + 2e) = n_2(l_2 + 2e)$$

$$\therefore \quad n_1 l_1 + 2e\,n_1 = n_2 l_2 + 2e\,n_2$$

$$\therefore \quad 2e\,n_1 - 2e\,n_2 = n_2 l_2 - n_1 l_1$$

$$\therefore \quad 2e(n_1 - n_2) = n_2 l_2 - n_1 l_1$$

$$\therefore \quad e = \frac{n_2 l_2 - n_1 l_1}{2(n_1 - n_2)}$$

18. In a resonance tube experiment a tuning fork resonates wth an air column 10 cm long and again resonates when, it is 32.2 cm long. Calculate the wavelength of wave and the end correction.

Given :

l_1 = 10 cm

l_2 = 32.2 m

To Find :

λ = ?

e = ?

Formula :

i) $L = l + e$

ii) $e = \dfrac{l_2 - 3l_1}{2}$

Solution :

$$L = l + e$$

$$\therefore \quad L_1 = l_1 + e = \frac{\lambda}{4} \quad ...(i)$$

$$L_2 = l_2 + e = \frac{3\lambda}{4} \quad ...(ii)$$

Subtract equation (i) from equation (ii)

$$\therefore \quad (l_2 + e) - (l_1 + e) = \frac{3\lambda}{4} - \frac{\lambda}{4}$$

$$\therefore \quad (l_2 - l_1) = \frac{\lambda}{2}$$

$$\therefore \quad \lambda = 2(l_2 - l_1)$$

$$= 2(32.2 - 10.0)$$

$$= 2(22.2)$$

$$\therefore \quad \lambda = 44.4 \text{ cm}$$

$$\text{Now, } e = \frac{l_2 - 3l_1}{2}$$

$$\therefore \quad e = \frac{(32.2) - 3(10)}{2}$$

$$\therefore \quad e = \frac{2.2}{2}$$

$$\therefore \quad e = 1.1 \text{ cm}$$